Exercise 8.1 (Revised) - Chapter 8 - Quadrilaterals - Ncert Solutions class 9 - Maths

Updated On 11-02-2025 By Lithanya

Chapter 8 - Quadrilaterals - NCERT Solutions for Class 9 Maths

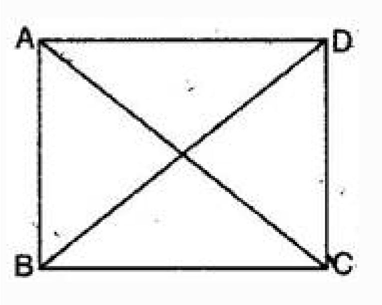
Ex 8.1 Question 1.

If the diagonals of a parallelogram are equal, show that it is a rectangle.

Answer.

Given: $ABCD$ is a parallelogram with diagonal $AC =$ diagonal BD

To prove: $ABCD$ is a rectangle.



Proof: In triangles ABC and ABD,
$\mathrm{AB}=\mathrm{AB}[$ Common $]$
$\mathrm{AC}=\mathrm{BD}[$ Given $]$
$\mathrm{AD}=\mathrm{BC}\,[$ opp. Sides of $\mathrm{a}\|_{\mathrm{gm}}]$
$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{BAD}[\mathrm{By}$ SSS congruency$]$
$\Rightarrow \angle \mathrm{DAB}=\angle \mathrm{CBA}[\mathrm{By}$ C.P.C.T. $]$

But $\angle \mathrm{DAB}+\angle \mathrm{CBA}=180^{\circ}$
$[\because AD \|_{\mathrm{BC}}$ and AB cuts them, the sum of the interior angles of the same side of transversal is $180^{\circ}]$

From eq. (i) and (ii),
$\angle \mathrm{DAB}=\angle \mathrm{CBA}=90^{\circ}$

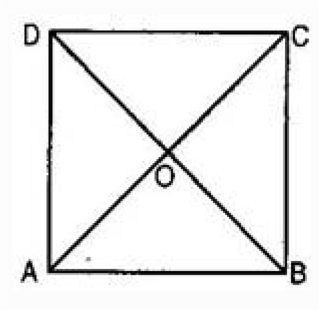
Hence $ABCD$ is a rectangle.

Ex 8.1 Question 2.

Show that the diagonals of a square are equal and bisect each other at right angles.

Answer.

Given: $ABCD$ is a square. AC and BD are its diagonals bisect each other at point O.



To prove: $AC = BD$ and $AC \perp BD$ at point O.

Proof: In triangles ABC and BAD,
$AB = AB$[Common]
$\angle ABC = \angle BAD = 90^\circ$
$BC = AD$[Sides of a square]
$\therefore \triangle ABC \cong \triangle BAD$ [By SAS congruency]
$\Rightarrow AC = BD$[By C.P.C.T.]Hence proved.

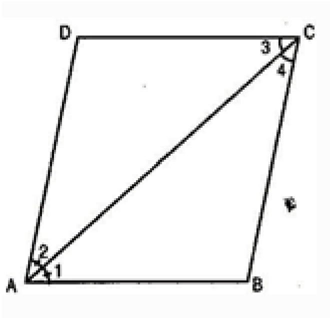
Now in triangles AOB and AOD,
$AO = AO$[Common]
$AB = AD$[Sides of a square]
$\therefore \triangle AOB \cong \triangle AOD$[By SSS congruency]
$\angle AOB = \angle AOD$[By C.P.C.T.]

But $\angle AOB + \angle AOD = 180^\circ$ [Linear pair]

But $\angle AOB + \angle AOD = 180^\circ$ [Linear pair]
$\therefore \angle AOB = \angle AOD = 90^\circ$
$\Rightarrow OA \perp BD$ or $AC \perp BD$ Hence proved.

Ex 8.1 Question 3.

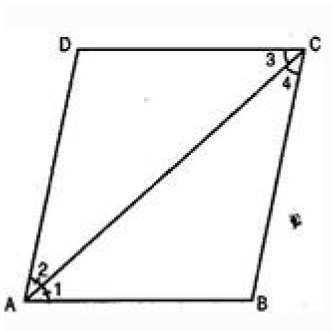
Diagonal AC of a parallelogram ABCD bisects $\angle \mathbf{A}$ (See figure). Show that:



(i) It bisects $\angle \mathbf{C}$ also.
(ii) ABCD is a rhombus.

Answer.

Diagonal AC bisects $\angle A$ of the parallelogram $ABCD$.



(i) Since $AB \| DC$ and AC intersects them.
$\therefore \angle 1 = \angle 3$ [Alternate angles]
Similarly $\angle 2 = \angle 4$
But $\angle 1 = \angle 2$ [Given]
$\therefore \angle 3 = \angle 4$ [Using eq. (i), (ii) and (iii)]

Thus AC bisects $\angle C$.
(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$
$\Rightarrow AD = CD$[Sides opposite to equal angles]
$\therefore AB = CD = AD = BC$

Hence $ABCD$ is a rhombus.

Ex 8.1 Question 4.

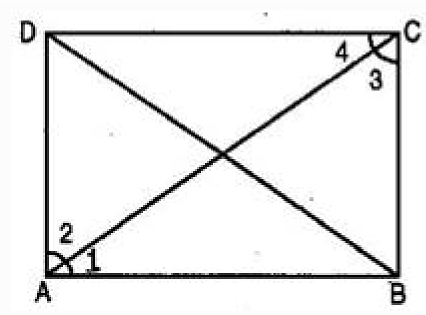
ABCD is a rectangle in which diagonal $\mathbf{AC}$ bisects $\angle \mathbf{A}$ as well as $\angle \mathbf{C}$. Show that:
(i) ABCD is a square.
(ii) Diagonal BD bisects both ∠ B as well as ∠ D.

Answer.

$ABCD$ is a rectangle. Therefore $AB = DC$ .... (i)

And $BC = AD$

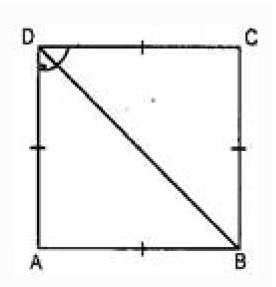
Also $\angle A = \angle B = \angle C = \angle D = 90^\circ$



(i) In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ADC}$
$\angle 1=\angle 2$ and $\angle 3=\angle 4$
[AC bisects $\angle \mathrm{A}$ and $\angle \mathrm{C}$ (given)]
$\mathrm{AC}=\mathrm{AC}$[Common]
$\therefore \Delta \mathrm{ABC} \cong \triangle \mathrm{ADC}$[By ASA congruency]
$\Rightarrow \mathrm{AB}=\mathrm{AD}$

From eq. (i) and (ii), $AB=BC=CD=AD$

Hence $ABCD$ is a square.



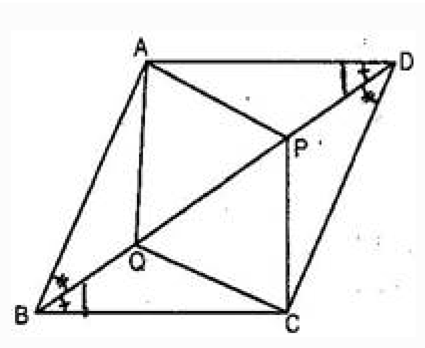
(ii) In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ADC}$
$AB=BA$[Since $ABCD$ is a square]
$AD=DC$[Since $ABCD$ is a square]
$\mathrm{BD}=\mathrm{BD}$ [Common]
$\therefore \Delta \mathrm{ABD} \cong \Delta \mathrm{CBD}$ [By SSS congruency]
$\Rightarrow \angle \mathrm{ABD}=\angle \mathrm{CBD}$ [By C.P.C.T.].........(iii)

And $\angle \mathrm{ADB}=\angle \mathrm{CDB}$[By C.P.C.T.] (iv)

From eq. (iii) and (iv), it is clear that diagonal BD bisects both $\angle \mathrm{B}$ and $\angle \mathrm{D}$.

Ex 8.1 Question 5.

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP=BQ$ (See figure). Show that:



(i) $\triangle$ APD $\cong$ $\triangle$ CQB
(ii) $\mathbf{AP}=\mathbf{CQ}$
(iii) $\triangle \mathrm{AQB} \cong \triangle \mathrm{CPD}$
(iv) $AQ=CP$
(v) APCQ is a parallelogram.

Answer.

(i) In $\triangle \mathrm{APD}$ and $\triangle CQB$,
$DP=BQ$[Given]
$\angle \mathrm{ADP}=\angle \mathrm{QBC}$ [Alternate angles (AD $\|_{\mathrm{BC}}$ and BD is transversal)]
$\mathrm{AD}=\mathrm{CB}$ [Opposite sides of parallelogram]
$\therefore \triangle \mathrm{APD} \cong \triangle \mathrm{CQB}$ [By SAS congruency]
(ii) Since $\triangle \mathrm{APD} \cong \triangle \mathrm{CQB}$
$\Rightarrow \mathrm{AP}=\mathrm{CQ}$[By C.P.C.T.]
(iii) In $\triangle \mathrm{AQB}$ and $\triangle \mathrm{CPD}$,
$\mathrm{BQ}=\mathrm{DP}$[Given]

$\angle \mathrm{ABQ}=\angle \mathrm{PDC}$ [Alternate angles (AB $\|^{\mathrm{CD}}$ and BD is transversal)]
$AB=CD$[Opposite sides of parallelogram]
$\therefore \triangle \mathrm{AQB} \cong \triangle \mathrm{CPD}$[By SAS congruency]
(iv) Since $\triangle AQB \cong \triangle CPD$
$\Rightarrow \mathrm{AQ}=\mathrm{CP}$[By C.P.C.T.]

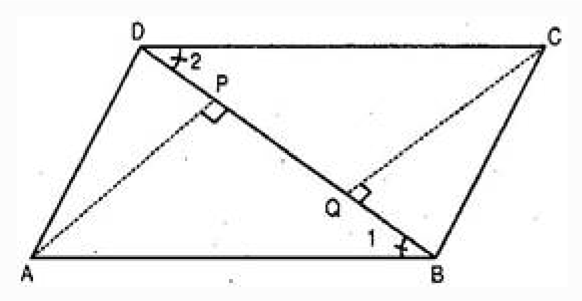
(v) In quadrilateral APCQ,
$\mathrm{AP}=\mathrm{CQ}$[proved in part (i)]
$\mathrm{AQ}=\mathrm{CP}$[proved in part (iv)]

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

Ex 8.1 Question 6.

ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:



(i) $\triangle$ APB $\cong \triangle CQD$
(ii) $\mathbf{AP}=\mathbf{CQ}$

Answer.

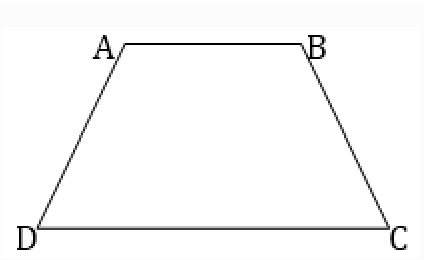
Given: ABCD is a parallelogram. $\mathrm{AP} \perp \mathrm{BD}$ and $\mathrm{CQ} \perp \mathrm{BD}$

To prove: (i) $\triangle \mathrm{APB} \cong \triangle CQD$ (ii) $\mathrm{AP}=\mathrm{CQ}$

Proof: (i) In $\triangle \mathrm{APB}$ and $\triangle CQD$,
$\angle 1=\angle 2$ [Alternate interior angles]
$\mathrm{AB}=\mathrm{CD}$[Opposite sides of a parallelogram are equal]
$\angle \mathrm{APB}=\angle \mathrm{CQD}=90^{\circ}$
$\therefore \triangle \mathrm{APB} \cong \triangle \mathrm{CQD}$[By ASA Congruency]
(ii) Since $\triangle \mathrm{APB} \cong \triangle CQD$
$\therefore AP=CQ[By\ C.P.C.T.$

Ex 8.1 Question 7.

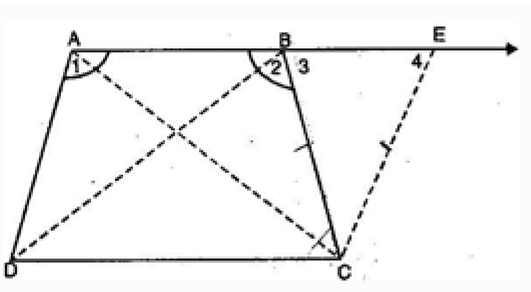
$ABCD$ is a trapezium in which $AB \| CD$ and $AD=BC$ (See figure). Show that:



(i) $\angle \mathrm{A}=\angle \mathrm{B}$
(ii) $\angle \mathbf{c}=\angle \mathrm{D}$
(iii) $\triangle \mathrm{ABC} \cong \triangle \mathrm{BAD}$
(iv) Diagonal AC $=$ Diagonal BD

Answer.

Given: $ABCD$ is a trapezium.
$AB \| CD$ and $AD=BC$



To prove: (i) $\angle \mathrm{A}=\angle \mathrm{B}$
(ii) $\angle \mathrm{C}=\angle \mathrm{D}$
(iii) $\triangle \mathrm{ABC} \cong \triangle \mathrm{BAD}$
(iv) Diag. $\mathrm{AC}=$ Diag. BD

Construction: Draw CE $\|$ AD and extend
AB to intersect CE at E.

Proof: (i) As AECD is a parallelogram.[By construction]
$\therefore AD=EC$

But $AD = BC$ [Given]
$\therefore BC = EC$
$\Rightarrow \angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now $\angle 1 + \angle 4 = 180^\circ$ [Interior angles]

And $\angle 2 + \angle 3 = 180^\circ$ [Linear pair]
$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2 [\because \angle 3 = \angle 4]$
$\Rightarrow \angle \mathrm{A} = \angle \mathrm{B}$
(ii) $\angle 3 = \angle$ C[Alternate interior angles]

And $\angle \mathrm{D} = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [$\triangle \mathrm{BCE}$ is an isosceles triangle]
$\therefore \angle C = \angle D$
(iii) In $\triangle ABC$ and $\triangle BAD$,
$\mathrm{AB} = \mathrm{AB}$ [Common]
$\angle 1 = \angle 2$ [Proved]
$\mathrm{AD} = \mathrm{BC}$ [Given]
$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{BAD}$[By SAS congruency]
(iv) We had observed that,
$\therefore \triangle \mathrm{ABC} \cong \triangle BAD$

$\Rightarrow \mathrm{AC} = \mathrm{BD}$ [By C.P.C.T.]

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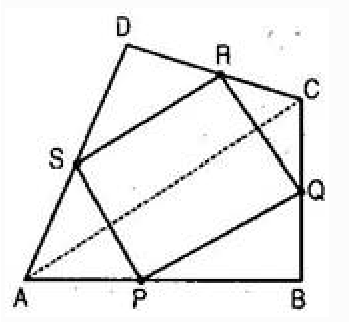
Ex 8.2 Question 1.

ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:



(i) SR AC and $SR = \frac{1}{2}\mathrm{AC}$
(ii) $\mathbf{PQ} = \mathbf{SR}$
(iii) PQRS is a parallelogram.

Answer.

In $\triangle ABC, P$ is the mid-point of AB and Q is the mid-point of BC.

Then $\mathrm{PQ} \| \mathrm{AC}$ and $\mathrm{PQ} = \frac{1}{2}\mathrm{AC}$
(i) In $\triangle ACD, R$ is the mid-point of CD and S is the mid-point of AD.

Then $SR \| AC$ and $SR = \frac{1}{2}\mathrm{AC}$
(ii) Since $PQ = \frac{1}{2}\mathrm{AC}$ and $SR = \frac{1}{2}\mathrm{AC}$

Therefore, $\mathrm{PQ} = \mathrm{SR}$
(iii) Since $PQ \| AC$ and $SR \| AC$

Therefore, PQ II SR [two lines parallel to given line are parallel to each other]

Now $PQ = SR$ and $PQ \| SR$

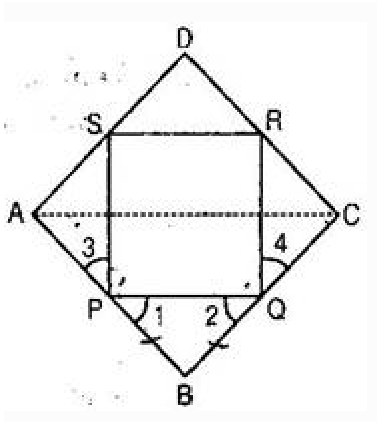
Therefore, PQRS is a parallelogram.

Ex 8.2 Question 2.

ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.

Answer.

Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.



To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In $\triangle ABC, P$ is the mid-point of AB and Q is the mid-point of BC.
$\therefore PQ \| AC$ and $PQ = \frac{1}{2}\mathrm{AC}$

In $\triangle ADC, R$ is the mid-point of CD and S is the mid-point of AD.
$\therefore SR \| AC$ and $SR = \frac{1}{2}AC$
From eq. (i) and (ii), $PQ^{\|}\mathrm{SR}$ and $\mathrm{PQ} = \mathrm{SR}$
$\therefore PQRS$ is a parallelogram.

Now $ABCD$ is a rhombus. [Given]
$\therefore AB = BC$
$\Rightarrow \frac{1}{2}\mathrm{AB} = \frac{1}{2}\mathrm{BC} \Rightarrow \mathrm{PB} = \mathrm{BQ}$
$\therefore \angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

$AP = CQ[P$ and Q are the mid-points of AB and BC and $AB = BC]$

Similarly, AS $= \mathrm{CR}$ and PS $= \mathrm{QR}$ [Opposite sides of a parallelogram]
$\therefore \triangle\mathrm{APS} \cong \triangle\mathrm{CQR}$ [By SSS congreuancy]
$\Rightarrow \angle 3 = \angle 4$ [By C.P.C.T.]

Now we have $\angle 1 + \angle\mathrm{SPQ} + \angle 3 = 180^\circ$

And $\angle 2 + \angle\mathrm{PQR} + \angle 4 = 180^\circ$ [Linear pairs]
$\therefore \angle 1 + \angle\mathrm{SPQ} + \angle 3 = \angle 2 + \angle\mathrm{PQR} + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]
$\therefore \angle\mathrm{SPQ} = \angle\mathrm{PQR}$

Now PQRS is a parallelogram [Proved above]
$\therefore \angle\mathrm{SPQ} + \angle\mathrm{PQR} = 180^\circ$
(iv) [Interior angles]

Using eq. (iii) and (iv),
$\angle\mathrm{SPQ} + \angle\mathrm{SPQ} = 180^\circ \Rightarrow 2\angle\mathrm{SPQ} = 180^\circ$
$\Rightarrow \angle\mathrm{SPQ} = 90^\circ$

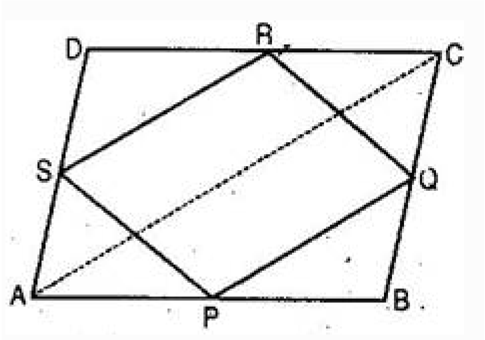
Hence $PQRS$ is a rectangle.

Ex 8.2 Question 3.

ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer.

Given: A rectangle $ABCD$ in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. $\mathrm{PQ}, \mathrm{QR}, \mathrm{RS}$ and SP are joined.



To prove: $PQRS$ is a rhombus.

Construction: Join AC.

Proof: In $\triangle\mathrm{ABC}, \mathrm{P}$ and Q are the mid-points of sides AB, BC respectively.
$\therefore PQ \| AC$ and $PQ = \frac{1}{2}\mathrm{AC}$
In $\triangle ADC, R$ and S are the mid-points of sides CD, AD respectively.

$\therefore SR \| AC$ and $SR = \frac{1}{2}\mathrm{AC}$
From eq. (i) and (ii), $PQ \| SR$ and $PQ = SR$
$\therefore$ PQRS is a parallelogram.

Now $ABCD$ is a rectangle. [Given]
$\therefore AD = BC$
$\Rightarrow \frac{1}{2}\mathrm{AD} = \frac{1}{2}\mathrm{BC} \Rightarrow \mathrm{AS} = \mathrm{BQ}$
In triangles APS and BPQ,
$AP = BP[P$ is the mid-point of $AB]$

$\angle \mathrm{PAS} = \angle \mathrm{PBQ}$ [Each 90°]

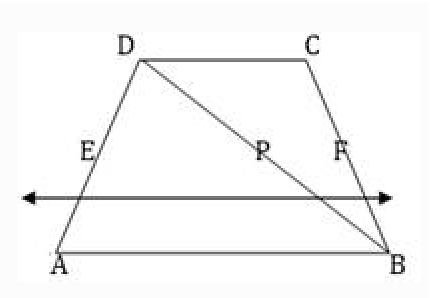
And $AS = BQ$[From eq. (iv)]
$\therefore \triangle \mathrm{APS} \cong \triangle \mathrm{BPQ}$ [By SAS congruency]
$\Rightarrow PS = PQ[By\ \mathrm{C.P.C.T.}]$

From eq. (iii) and (v), we get that PQRS is a parallelogram.
$\Rightarrow PS = PQ$
$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

Ex 8.2 Question 4.

ABCD is a trapezium, in which $AB \| DC, BD$ is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Answer.

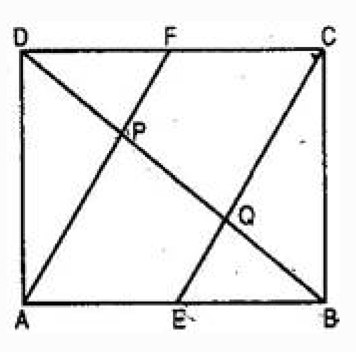
Let diagonal BD intersect line EF at point P.

In $\triangle \mathrm{DAB}$,
E is the mid-point of AD and $EP \| AB[\because EF \| AB$ (given) P is the part of $EF]$
$\therefore P$ is the mid-point of other side, BD of $\triangle DAB$.
[A line drawn through the mid-point of one side of a triangle, parallel to another side intersects the third side at the mid-point]

Now in $\triangle BCD$,
P is the mid-point of BD and $PF \| DC[\because EF \| AB$ (given) and $AB \| DC$ (given)]
$\therefore EF \| DC$ and PF is a part of EF.
$\therefore F$ is the mid-point of other side, BC of $\triangle BCD$. [Converse of mid-point of theorem]

Ex 8.2 Question 5.

In a parallelogram $ABCD, E$ and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Answer.

Since E and F are the mid-points of AB and CD respectively.
$\therefore AE = \frac{1}{2}\mathrm{AB}$ and $CF = \frac{1}{2}$CD..........(i)

But $ABCD$ is a parallelogram.
$\therefore AB = CD$ and $AB \|_{DC}$
$\Rightarrow \frac{1}{2}\mathrm{AB} = \frac{1}{2}\mathrm{CD}$ and $\mathrm{AB} \|_{\mathrm{DC}}$
$\Rightarrow AE = FC$ and $AE \|^{FC}$[From eq. (i)]
$\therefore \mathrm{AECF}$ is a parallelogram.
$\Rightarrow$ FA \| CE $\Rightarrow$ FP \| CQ [FP is a part of FA and CQ is a part of CE]

Since the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\triangle DCQ$, F is the mid-point of CD and $\Rightarrow FP \| CQ$
$\therefore P$ is the mid-point of DQ.
$\Rightarrow DP = PQ$
Similarly, In $\triangle ABP$, E is the mid-point of AB and $\Rightarrow EQ \| AP$

$\therefore Q$ is the mid-point of BP.
$\Rightarrow BQ = PQ$
From eq. (iii) and (iv),
$DP = PQ = BQ \ldots\ldots\ldots (v)$

Now $BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$
$\Rightarrow BQ = \frac{1}{3}BD$
From eq. (v) and (vi),
$DP = PQ = BQ = \frac{1}{3}BD$
$\Rightarrow$ Points P and Q trisects BD.

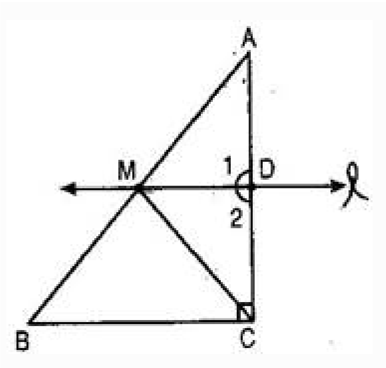
So AF and CE trisects BD.

Ex 8.2 Question 6.

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

Answer.

(i) In $\triangle ABC$, M is the mid-point of AB [Given]



$MD \| BC$
$\therefore AD = DC$ [Converse of mid-point theorem]

Thus D is the mid-point of AC.
(ii) $l \| BC$ (given) consider AC as a transversal.
$\therefore \angle 1 = \angle$ C [Corresponding angles]
$\Rightarrow \angle 1 = 90^\circ$ $[\angle C = 90^\circ]$

Thus MD $\perp$ AC.
(iii) In $\triangle AMD$ and $\triangle CMD$,
$AD = DC$ [proved above]
$\angle 1 = \angle 2 = 90^\circ$ [proved above]
$MD = MD$ [common]
$\therefore \triangle AMD \cong \triangle CMD$ [By SAS congruency]
$\Rightarrow AM = CM$ [By C.P.C.T.]

Given that M is the mid-point of AB.

$\therefore AM = \frac{1}{2}AB$
From eq. (i) and (ii),
$CM = AM = \frac{1}{2}AB$